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journal homepage: www.elsevier.com/locate/eerThe effects of “nonbinding” price floors[☆]Stephen Salant^{b,*}, William Shobe^c, Neslihan Uler^{a,b}^a University of Maryland, College Park, MD, USA^b University of Michigan, Ann Arbor, MI, USA^c University of Virginia, Charlottesville, VA, USA

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ABSTRACT

Governments often impose price floors to protect sellers against low prices in markets characterized by uncertainty. Hard floors arise in grain and currency markets whenever the government acquisitions at the support price are unconstrained. Soft floors arise whenever such acquisitions are subject to a binding constraint. An important special case of a soft floor arises in cap-and-trade programs when the permit auction has a reserve price. Such an auction is equivalent to one with no reserve price combined with a government program to acquire at the reserve price up to the amount auctioned. Using a unified approach, we investigate a novel characteristic of such floors. Although seemingly “nonbinding,” such floors can influence the new equilibrium price. Whether inserted below or above the *laissez-faire* price, they can result in a new price strictly exceeding the floor—a phenomenon we dub “action at a distance.” We explain it theoretically and illustrate with simulations.

1. Introduction

Price floors are ubiquitous. They are used to support prices in grain markets, in currency markets, and in emissions permit markets. It is commonly believed that a floor inserted below the prevailing competitive price will have no effect and that a floor inserted above that price will raise the price to the level of the floor. Such beliefs are well-founded if the floor is a minimum wage and the price is the wage in a competitive labor market. But in the case of markets for storables like grain, currency, or emissions permits, the current price is connected to future prices and static supply and demand analysis is misleading.

Such markets are not static and extreme care must be exercised in using static supply and demand curves to analyze them. In these markets, the equilibrium current price typically depends on prices expected to prevail in the future. As we will show, the unanticipated imposition of a price floor below the current price may cause the current price to jump up because the policy raises the expected price in the future. The same phenomenon may occur if a floor is inserted above the current price. In that case, the new equilibrium may again strictly exceed the floor.

Our analysis encompasses two different types of floors: hard floors and soft floors. When the government stands ready to purchase unlimited amounts at the support price, the policy is referred to as a “hard floor”. Hard floors are used in grain and currency markets.

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A government bufferstock manager stands ready to buy whatever is offered at the floor price, and consequently the market price never falls below the hard floor.

If the manager stands ready to buy only a limited amount per period at the floor price, the market price can fall below the floor and the floor is dubbed “soft”. Constraints on government acquisitions arise when the support program has a limited budget or limited ability to reach remote areas as is often the case in developing countries¹ or when the government extends a loan equal to the dollar value of the current crop, priced at f per unit. The crop is used as collateral and is forfeited whenever the farmer would get less than f per unit if he put his crop on the market.² Soft floors also arise in cap-and-trade programs where emissions permits are auctioned subject to a reserve price.

In cap-and-trade programs to limit emissions of pollutants like CO₂, regulated agents must relinquish permits for each unit of pollutant emitted. Replacement permits are injected into the market through periodic permit auctions and the reserve price in such auctions is referred to as a “soft floor”. The reserve price is the floor on bids the government will entertain. An auction reserve price is a feature of many emission trading programs, including California’s cap-and-trade program (AB-32) and the Regional Greenhouse Gas Initiative (RGGI) as well as federal cap-and-trade proposals. While not yet a feature of the European Union Emission Trading System (EU-ETS), which exhibited long periods with lower than expected prices, many economists have argued for adoption of an auction reserve price without the benefit of extensive theoretical analysis of such a policy. The auction reserve price is called a “soft floor” because, in contrast to the hard floor case, the market price can fall below the floor. A policy where permits are auctioned without a reserve price f but then the government offers to buy back at price f per permit up to the number of permits auctioned is equivalent to an auction with a reserve price f .³

In this paper, we investigate theoretically the effect of each type of floor in the infinite-horizon stochastic, competitive storage model adopted independently by agricultural and environmental economists. This canonical model describes the stochastic evolution of the price of the storable good relative to a “background good”.⁴ Although a hard floor, a soft floor and *laissez faire* seem to be qualitatively dissimilar policies, we develop a unified framework to analyze them. Each can be regarded as special case of a government offer to buy at the floor price up to some maximum amount. In the case of *laissez faire*, the maximum is zero; in the case of a hard floor, the maximum is infinite; and in the case of a soft floor, the maximum is a finite amount (and equals to the amount auctioned in the case of emissions permit markets).

We find that, even if the floor is inserted below the initial market price, the price will, under specified conditions, jump up. Moreover, the jump is larger with a hard floor than with a soft floor. If the floor is inserted above the initial market price, the price may also jump up above the floor. We refer to this phenomenon where the equilibrium price strictly exceeds the floor but is nonetheless determined by the floor as “action at a distance”.

The intuition for our theoretical results can be summarized in the following way. Take the case of a hard floor in a grain market. Prior to imposition of the hard floor, farmers may be storing part of their crop to sell in the future and selling the remainder at the current price. To portray the current price as equating supply and demand, the analyst must therefore subtract from current supply the quantity of grain carried into the future. But if a “nonbinding” hard floor is then imposed below the current price, it may be binding in the future. It will, therefore, protect farmers against the price of their stored crop falling below the floor in some adverse future states of the world. As a result, farmers will carry more into the future, the current supply curve will shift further to the left, and the current price will rise. A soft floor may cause a smaller jump in the current price than a hard floor because it does not provide complete protection against the price falling below the floor in the future in some states of the world. The same mechanism explains why a floor inserted above the *laissez-faire* price may result in the new equilibrium price exceeding the floor.

Seemingly nonbinding hard floors affect real-world grain markets. The *Financial Times* (August 19, 2013) reports “Beijing’s minimum procurement price for domestic long grain rice is set at \$420 per tonne but spot prices are about \$600 per tonne...” Nonetheless, without specifying the mechanism, *FT* goes on to say that “Beijing’s price support for grain has led to high prices”.

The emissions trading market in California aptly illustrates how a soft floor achieves action at a distance if and only if it raises expected future prices. Because of the involvement of the courts, the increase in the expected future price caused by the soft floor was first switched off and then turned back on again, resulting in the price effects we predict. In particular, prior to 2016, the floor was expected to persist and action at a distance occurred with the auctions fully subscribed and prices settling above the reserve price. In 2016, however, a legal challenge from the California Chamber of Commerce threatened to invalidate the entire program. For the four quarterly auctions of 2016 and the first auction of 2017, legal uncertainty suddenly rendered the static analysis appropriate. With the continuation of the program in jeopardy, the reserve price of future auctions no longer provided protection against a future capital loss, and traders acquired only the permits needed to cover their emissions at the next true-up. As a result, each of the 5 auctions settled at the reserve price, with every auction under-subscribed. On June 28 2017, however, the legal risk subsided and

¹ In India, for example, Reuters (2020) reports that there is a minimum support price for rice and for wheat but most farmers cannot access these supports and sell instead on the market at a discount of 25%–35%.

² In “Non-recourse Marketing Assistance Loans” farmers in the US can borrow using collateral valued at an exogenous price f . “Non-recourse” means the government is obliged to accept the collateral instead of repayments of the loan even if the market value of the collateral is less than what was borrowed. Forfeiting the collateral is optimal whenever the market price falls below f . When the random shocks affect demand and the harvest is deterministic, the soft floor can be interpreted as representing this policy.

³ As shown in Section 2.4, the two mechanisms are equivalent in the relevant respects: both result in the same market price and private stock. One difference of no relevance to the equilibrium is that when the market price is strictly below the floor ($p < f$), the auction with the buyback confers on private holders of g permits (not necessarily agents who purchased permits at the auction) a rent of $f - p$ per permit.

⁴ For a general equilibrium formulation that takes explicit account of the background good, see Appendix A.

continuation of the program was assured.⁵ Future price floors then resumed their role as protection against future capital losses, carryovers increased, and the price jumped again above the price floor, a situation which has persisted ever since.⁶ In their study of the California cap-and-trade program, Borenstein et al. (2019) observed that “the fact that California’s allowance prices were higher than the other major GHG cap-and-trade programs from its inception. . . is almost certainly due to its relatively high price floor”. Our paper provides conditions sufficient for what Borenstein et al. (2019) sensed was occurring.

Both the literature on agricultural price supports and the literature on emissions permits analyze the stochastic, competitive equilibrium that arises when profit-maximizing private agents can sell an asset (be it a bushel of wheat or a permit to emit a ton of carbon) at the current price or can store it for future use. Both literatures recognize that unpredictable weather and other shocks continually disturb these markets.

Competitive equilibrium carryover models of grain under harvest uncertainty were first solved by Gustafson (1958) and subsequently by Samuelson (1971). Both solved the associated planning problem using dynamic programming and then appealed to the second welfare theorem to infer equilibrium price functions. Important subsequent contributions were made by Kohn (1978), Scheinkman and Schechtman (1983), and Deaton and Laroque (1992), among others.

Governments often intervene in agricultural and other markets by imposing a “price band”, characterized by a price ceiling and a price floor. The price floor is enforced by unlimited government purchases at the support price. The price ceiling is enforced by sales at the ceiling up to the limit of the government’s stockpile. Realizing that nothing may be maximized in markets distorted by government policy, Gardner abandoned the dynamic-programming approach and deduced the equilibrium price functions recursively on a computer. This required him to specify numerically the demand curve, harvest distribution, the price ceiling and the price floor.⁷ Following Gardner, there have been many subsequent simulations, most notably Miranda and Helmberger (1988) and Williams and Wright (1991), Wright and Williams (1988).⁸

In a deterministic setting, Lee (1978) used an exhaustible resource model to show how a hard price ceiling that would only bind at some future date would lower the current prices of a resource, the first example of a government price policy causing action at a distance.

By contrast, in the stochastic case, a price floor will often affect the current equilibrium price even if, in many possible realizations, the floor is never binding. This case is particularly relevant to price floors in emission markets. Schennach (2000) was among the first to note the close connection between the models of commodity markets and the modeling of bankable allowances under uncertainty about future allowance demand. She observed that an unanticipated increase in the possibility that the bank might fall to zero at some point in the future (a “stockout”) raises the current price. Schennach pointed out that the current-period effect of the possibility of future stockouts had been extensively analyzed in the commodity markets literature.

In simulating the effects of *price ceilings* in emission markets, Burtraw et al. (2010) observed that “even if the one-sided safety valve never does bind, its introduction to a cap and trade program affects the expectation of future emissions levels and allowance prices and thereby the expectations about the payoff from various investment strategies” Burtraw et al. (2010, pp. 4922). They then showed by simulation how a ceiling price affects the probability density of future prices.

In experiments on price collars in the EU-ETS, Holt and Shobe (2016) noted that, even when the auction reserve price was strictly below the spot price, raising this soft floor raised the observed spot price, and that this effect extended to cases where the price floor was never binding in any period during the session.

While there is a general recognition that price floors control downside price uncertainty in emission markets (Hintermayer, 2020) as well as in commodity markets generally, a theoretical understanding of the underlying mechanism clarifies that such floors may exert action at a distance even when inserted *above* the no-policy price. In addition, our analysis clarifies why distinguishing between soft and hard floors is important when simulating, estimating, or exploring experimentally the effects of price containment policies.

Simulations and experiments can be extremely valuable in uncovering surprising phenomena, but without the guidance of theory, each methodology has serious limitations. First, one never knows whether the phenomena uncovered would disappear if different numerical parameters of the demand curve or shock distribution had been used. Second, neither methodology is well-suited to identifying different phenomena generated by the same mechanism.

Theory suffers from neither limitation. For example, we prove analytically that the phenomena we discuss hold over a broad set of demand curves and shock distributions. Moreover, once one understands theoretically why a soft floor inserted below the *laissez-faire* price causes action at a distance, it becomes clear that a soft floor inserted *above* that price would have a similar effect and that it would again be amplified if the floor were hard instead of soft.

In this paper, we establish these results theoretically and conduct simulations to illustrate them. A companion paper, Salant et al. (2022), formulates a two-period model with characteristics similar to this infinite-horizon formulation, designs a laboratory experiment, and uses it to investigate whether subjects behave as theory predicts.

⁵ As recounted in an EDF-IETA report, “2017 brought greater market certainty as the California Third District Court of Appeal upheld the design of the cap-and-trade auctions, the California Supreme Court declined [on June 28, 2017] to review the case which upheld the lower court’s ruling, and the California Legislature passed a bill to extend the program until at least 2030. Taken together, these factors have created a great deal more assurance in the future of the program, and allowance prices and demand have steadily risen through the final three joint auctions of 2017”.

⁶ After the first few auctions, the price in the secondary market at the time of the auction was virtually the same as the price successful bidders paid for the identical permits at that auction. Auctions occur quarterly while trading on the secondary market occurs daily.

⁷ Unlike Gardner (1979) and Salant (1983), Wright and Williams (1982) introduced price-responsive supply into the model.

⁸ Our focus in this paper is on the immediate effects on the price of imposing hard or soft floors. Wright and Williams, on the other hand, do not study soft floors and focus instead on how hard floors affect the stationary distribution of prices. Hence, for the hard floor case, the analyses are complementary.

We contribute to two distinct literatures. As our literature review suggests, our results can help guide future analysis of the use of price floors and ceilings in cap-and-trade programs. And although agricultural economists are familiar with the long-run effects of hard price floors from simulations, we provide analytical results about the immediate effects of both soft and hard floors. Moreover, we do so using a contraction-mapping technique that can be applied whenever governments intervene in markets with carryovers.

We proceed as follows. In Section 2, we investigate the traditional infinite-horizon model. We show how to treat the three policy regimes in a unified manner and, use that approach to establish our theoretical results. Sub-Section 2.4 clarifies how our results can be re-interpreted when the shocks shift the demand curve, and should be of particular interest in the cap-and-trade application. In Section 3, we present our simulations. Section 4 concludes the paper.

2. The model

2.1. A unified framework for studying three distinct policies

We consider an infinite-horizon stochastic competitive model of storage where “speculators” are risk neutral and have rational expectations. To avoid confusion, we present the agricultural interpretation first and the emissions-trading interpretation in Section 2.4. The analysis is in discrete time.⁹ In the agricultural interpretation, a period represents the time interval between harvests.

We assume that in every period the mean harvest g units is displaced by one of K possible exogenous shocks denoted α_k , with $\alpha_1 > \dots > 0 > \dots > \alpha_K$. State k occurs with exogenous probability $\pi_k > 0$. We assume $\sum_{k=1}^K \pi_k \alpha_k = 0$.

After observing the realized current available stock, agents decide how much of the stock to carry over and how much to sell to consumers or the government. Carryover from period t is denoted x_t . We assume no storage costs, so what is carried out of period t equals what is carried into period $t + 1$. This carryover is supplemented at the beginning of period $t + 1$ by the realized harvest $g + \alpha_k$. Consumer demand is a strictly decreasing, differentiable function of the current price with choke price p^c . That is, $D(p) = 0$ for $p \geq p^c$ and $D(p) > 0, D'(p) < 0$ for $0 < p < p^c$.

We wish to compare the equilibrium that arises under three distinct government policies: no price floor, a hard price floor, and a soft price floor. The soft price floor arises whenever there is a binding constraint on what the government can purchase at the support price. Under agricultural interpretation, soft floors can arise when government grain purchases are budget-constrained or when crops are used as collateral in non-recourse loans.

Hard floors, soft floors, and no floors are special cases of a more general policy. Under that policy, the government defends an exogenous price floor (f) by buying nothing if the market price exceeds the floor, \bar{g} if the market price is below the floor and some intermediate amount if the market price coincides with the floor.¹⁰ We denote the government demand correspondence as $R(p_t; \bar{g})$:

$$R(p_t; \bar{g}) \begin{cases} = 0 & \text{if } p_t > f \\ \in [0, \bar{g}] & \text{if } p_t = f \\ = \bar{g} & \text{if } p_t < f. \end{cases} \tag{1}$$

For no floor (denoted by a superscript N), $\bar{g} = 0$. For the soft floor (denoted by a superscript S), $\bar{g} = g$, the expected harvest. For the hard floor (denoted by a superscript H), $\bar{g} = \infty$. Henceforth, we suppress f and \bar{g} and refer to the government demand correspondence in the three policy regimes as $R^i(p_t^i)$ for $i = N, S, H$. Let $A = x_{t-1} + g + \alpha_k$ denote the realized availability at time t . When we express price or carryover as a function of availability, we write $p^i(A)$ or $x^i(A)$, respectively, suppressing the dependence on the floor that exists when $i = H, S$. The existence, uniqueness, continuity and other properties of $p^H(A)$ and $p^S(A)$ are established in Appendix B. $p^N(A)$ has been studied extensively in the literature.

In any infinite-horizon equilibrium the market clears at each date and state, and each price-taking, risk-neutral agent maximizes expected wealth. Hence, given policy regime i , any equilibrium price function $p^i(A)$ and storage function $x^i(A)$ must satisfy the following conditions:

$$A = x^i(A) + D(p^i(A)) + R^i(p^i(A)) \tag{2}$$

$$x^i(A) \geq 0, p^i(A) - \beta \sum_{k=1}^K \pi_k p^i(x^i(A) + g + \alpha_k) \geq 0 \text{ with comp. slackness} \tag{3}$$

Eq. (2) requires that, for any realized availability A in the current period, the price adjusts to clear the market. Eq. (3) requires that, for any availability A , price-taking, risk-neutral speculators with common probability assessments of the state next period carry over a quantity which maximizes expected profits. If carrying stock over is less profitable than an immediate sale, none is carried over; if positive carryovers occur, a sale in the future is expected to be as profitable as an immediate sale.

⁹ In footnote 17, we briefly discuss the continuous-time limit.

¹⁰ This formulation is general enough to also capture soft floors in the emissions permit markets. In that case, set \bar{g} equal to the number of permits auctioned. We are indebted to Christophe Gouel for this insight. For more details on the cap-and-trade interpretation, see Section 2.4.

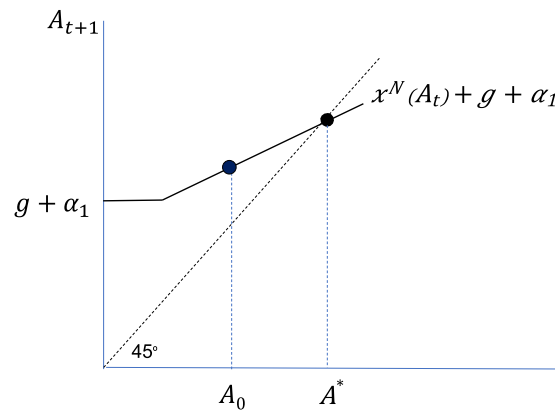


Fig. 1. Determination of fixed point A^* when initial carryover is positive ($x^N(A_0) > 0$).

2.2. An unnoticed implication of the model under *laissez-faire*

To begin, we review results established by Gustafson, Samuelson, Wright, Williams, and others for the *laissez-faire* version of the infinite-horizon problem (denoted by a superscript N). In the special case of Eqs. (2)–(3) where $R^N(p_t) = 0$, these authors have shown that the unique equilibrium price function $p^N(A)$ is strictly positive and strictly decreasing while the equilibrium carryover is zero if availability is below some level and is strictly positive with $\frac{dx^N}{dA} \in (0, 1)$ if availability is larger. Newbery and Stiglitz (1982) provides a succinct treatment of these results.

Since these price and carryover functions generate a competitive equilibrium, they will result in market-clearing starting with any initial availability and moving forward for every sequence of shocks that can occur. To show that these functions no longer generate an equilibrium once a floor of either type is inserted below the current *laissez-faire* price, we must find a sequence of shocks that can occur with positive probability and that would result in the market failing to clear at some future date if this pair of functions were followed.

The stock available for consumption or storage in period $t + 1$ equals the carryover from period t plus the stochastic harvest realized at the outset of $t + 1$. Recall that α_1 denotes the largest of the K possible harvest shocks. Let A^* denote the unique solution to the following equation:

$$A^* = x^N(A^*) + g + \alpha_1. \tag{4}$$

We depict the determination of A^* in Fig. 1.¹¹ If $A_0 < A^*$, availability resulting from a run of largest harvests will follow the following deterministic difference equation and will, therefore, monotonically increase toward A^* :

$$A_{t+1} = x^N(A_t) + g + \alpha_1. \tag{5}$$

We now show that a seemingly nonbinding price floor can displace the equilibrium in the *laissez-faire* model of the literature by showing a sequence of shocks that will lead to a disequilibrium if the *laissez-faire* price and carryover functions are followed.

Proposition 1. *If $x^N(A_0) > 0$ and $A_0 < A^*$ then any seemingly “nonbinding” price floor $f \in (p^N(A^*), p^N(A_0)]$ will displace the equilibrium.*

Proof. Choose any \hat{A} in the interval $[A_0, A^*)$. Then since the price function is strictly decreasing, $p^N(A^*) < p^N(\hat{A}) \leq p^N(A_0)$. Define $f = p^N(\hat{A})$. Then f is equal to or strictly below the initial price. In the latter case, it appears “nonbinding”. Nonetheless, a run of large harvests can occur with positive probability. In that case, A_t will grow monotonically toward A^* . Consider the earliest period when $A_t > \hat{A}$. In the absence of government intervention, the market would clear. But with either type of floor, the additional demand of the government buffer stock manager ($R^H(p_t) > 0$ and $R^S(p_t) > 0$ since $p < f$) would result in excess demand. Hence, the nonbinding floor displaces the equilibrium. \square

More generally, in any stochastic model, imposition of an unanticipated but permanent policy may not appear to bind initially but will nonetheless displace the equilibrium if there is any future state achievable with positive probability where the market would no longer clear after the policy is imposed.

¹¹ Fig. 1 in Newbery and Stiglitz (1982) provides an alternate treatment of the evolution of availability with successive maximal harvests.

2.3. A sufficient condition for a strict ranking of prices under the three policies

How is equilibrium restored in our model? Intuitively, since the carryover in the original equilibrium is strictly positive in every period, the initial price in the absence of the government intervention must equal the discounted price initially expected to prevail in the future (Eq. (3)). Hence the excess demand in that future period will communicate itself to the price in the initial period, resulting in an upward price jump. We establish this formally in Proposition 2, which provides sufficient conditions for an unanticipated floor inserted strictly below the *laissez-faire* price in an infinite-horizon competitive problem to cause a strict jump in the equilibrium price with the jump strictly larger if the floor is hard than if it is soft.

Proposition 2. *Under each of the three policies, there exists a unique equilibrium price function $p^i(A)$, which is continuous. The price functions can be weakly ranked across the three policies: $p^H(A) \geq p^S(A) \geq p^N(A)$. If the following conditions hold: (i) $x^N(\bar{A}) > 0$; (ii) $p^N(\bar{A}) > f$; (iii) $p^S(x^S(\bar{A}) + g + \alpha_1) < f$, then the rankings are strict: $p^H(\bar{A}) > p^S(\bar{A}) > p^N(\bar{A}) > f$ and $x^H(\bar{A}) > x^S(\bar{A}) > x^N(\bar{A})$.*

Proof. We sketch the proof here and refer the reader to Appendix B for details. If one knows the price as a function of availability in the next period, one can deduce the price as a function of availability in the current period using the complementary-slackness condition in (3), the clearing of the current market in (2), and the definition of $R^i(p^i)$ for $i = N, S, H$ in (1). Denote this mapping from a weakly decreasing price function next period to a weakly decreasing price function one period earlier under policy i as $T^i[\cdot]$ for $i = N, S, H$, where $[\cdot]$ denotes the price function in the operator's domain. We establish that T^i is a contraction mapping and hence there exists a unique fixed point for each policy, a weakly decreasing continuous function of availability. Denote this price function, $p^i(A)$ for $i = N, S, H$. Each fixed point can be approximated by applying T^i iteratively starting with any continuous, bounded, decreasing function.¹²

Next we note that if our initial price functions (denoted by a subscript 0) are weakly ranked as $p_0^H(A) \geq p_0^S(A) \geq p_0^N(A)$ that $T^H[p_0^H(A)] \geq T^S[p_0^S(A)] \geq T^N[p_0^N(A)]$. But these are again just three weakly ranked price functions and we know that if we apply T^H to the function on the left, T^S to the middle function and T^N to the function on the right, we will once again produce three weakly ranked price functions. . . It follows that the fixed points will be weakly ranked: $p^H(A) \geq p^S(A) \geq p^N(A)$.

The final step in the proof of Proposition 2, explained intuitively below, is to show under conditions (i)–(iii), that the discounted price expected next period, if the current available stock is \bar{A} , can be strictly ranked. This in turn implies that the current prices can be strictly ranked with the floor strictly below even the lowest of them. Since the government would then not be called upon to intervene under any of the three policies, the carryovers must be strictly ranked in the same order: $x^H(\bar{A}) > x^S(\bar{A}) > x^N(\bar{A})$. □

We provide the intuition for the rankings of the price and carryover functions to be strict. Suppose, given current availability \bar{A} , that the largest harvest would drive the price next period strictly below f , if a soft floor were in place (condition (iii)). Then if the floor were instead hard, the price next period would be strictly larger than the soft-floor price since it can never fall below the floor. And if *laissez faire* prevailed, the price next period would plausibly be strictly lower than the soft-floor price, since the government would never buy up g units. Since the price under the three policies can be weakly ranked in every state and strictly ranked in state 1, next period's discounted expected price under the three policies can be strictly ranked. And since condition (i) along with Eqs. (1) and (3) imply that the current price under each policy must equal the discounted expected price in the next period, the current price under each policy can be strictly ranked, with even the lowest of these, the *laissez-faire* price, strictly above the floor (condition ii).

Corollary 2.1. *If the conditions of Propositions 1 and 2 hold, then there exists a neighborhood above the *laissez-faire* price in which imposition of either a hard floor or a soft floor will cause the equilibrium price to jump above that floor.*

Proof. Suppose the conditions of Propositions 1 and 2 hold.¹³ Define $\hat{A} > \bar{A}$ such that $p^N(\hat{A}) = f$. Then we know that $p^i(\hat{A}) \neq p^N(\hat{A})$ for $i = H, S$ since otherwise $p^N(\hat{A})$ would remain an equilibrium when the floor was imposed, contradicting Proposition 1. Moreover, we know that $p^i(\hat{A}) > p^N(\hat{A})$ for $i = H, S$. For, otherwise when A is in the left neighborhood of \hat{A} , continuity would require that $p^i(A) < p^N(A)$ for $i = H, S$ which violates the weak ranking in Proposition 2.

Since the price functions are continuous, and at \hat{A} and the neighborhood to its left the equilibrium price with either type of floor strictly exceeds that floor, there must be a neighborhood to the right of \hat{A} where the equilibrium price functions with either type of floor also strictly exceed that floor, even though in the neighborhood to the right of \hat{A} the floor strictly exceeds the *laissez-faire* price since p^N is strictly decreasing. □

These results are proved for a “neighborhood” on either side of \hat{A} , the availability that makes the *laissez-faire* price just equal to the floor. The reader should keep an open mind about the width of the interval of availabilities sufficient for the occurrence of action at a distance. In Section 3, we show that this interval can be wide.

¹² Since the contraction-mapping approach to deducing the equilibrium when the government intervenes in a carryover model was explicated in Salant (1983), we have relegated a proof of Proposition 2 to Appendix B.

¹³ The assumptions in the two propositions are consistent. Given f , define \hat{A} so that $p^N(\hat{A}) = f$. Then since $p^N(A)$ is strictly decreasing, condition (ii) of Proposition 2 implies $\bar{A} < \hat{A}$, condition (iii) and the implied weak ranking $p^S(A) \geq p^N(A)$ together imply $\bar{A} < x^S(\bar{A}) + g + \alpha_1$, and stability implies that $\bar{A} < x^S(\bar{A}) + g + \alpha_1 \leq A^*$ where A^* is the fixed point in Proposition 1.

These two propositions clarify how the deterministic result of Lee (1978) generalizes to a stochastic setting with a hard floor. Lee considered the effects of imposing in a deterministic Hotelling model an unanticipated, permanent hard ceiling strictly above the current price (but below the choke price), implemented by the government's offer to sell from its bufferstock a perfect substitute at the ceiling price. Although it is therefore seemingly nonbinding, Lee showed that the unanticipated imposition of such a ceiling would result in the initial price jumping down. Neither Lee (nor others establishing a similar result for hard backstop technologies) consider the soft-ceiling case and hence they do not compare hard and soft ceilings.¹⁴ We show how hard and soft floors differ. Soft floors are important not merely because they arise in emission permit auctions but because they extend in an obvious way to cases like India's "minimum support price" where the annual budget of the government buying authority constrains the amount that can be purchased—the most common situation in developing countries where price supports abound.

2.4. Application to the cap-and-trade programs

The most important application of a soft floor is to the reserve price auctions in cap-and-trade programs to limit emissions of pollutants like CO₂. In such applications, a "period" is the length of time between auctions. The shock is assumed to affect demand, not supply. In particular, the permits that must be surrendered at the periodic true-up cover emissions since the last true-up and are represented by the demand curve $D(p^j)$ displaced by a random shock, α_k . The shock reflects the vagaries of the weather and other transient factors affecting the usage of air conditioning and furnaces. Instead of describing the case where a positive additive shock (α_k) shifts the supply curve to the right against a fixed demand curve, causing the market-clearing price to fall, our model could as easily describe the case where a negative shock of the same magnitude shifts the demand curve to the left ($D(p^j) - \alpha_k$) against a fixed supply curve, causing exactly the same reduction in price. The mean number of permits which must be relinquished, $D(p^j)$, is strictly decreasing because an increase in the permit price between true-ups would induce more abatement and fewer permits would have to be surrendered.

In cap-and-trade programs governing CO₂ emissions, g permits are auctioned. In the US, (but not EU ETS) these auctions have a reserve price of r . If the market price strictly exceeds the reserve price ($p > r$), bids at the auction will exceed the reserve price and all g permits will be auctioned. If, however, the market price is strictly below the reserve price ($p < r$) then no one would bid more than the market price, no bids would be accepted and no permits would be auctioned.

Precisely the same market price and availability would arise if instead the government auctioned g permits with no reserve price but also implemented a price support program with a soft floor set at the reserve price ($f = r$) and the maximum buyback set equal to the amount auctioned ($\bar{g} = g$). In that case as with a reserve price, g permits would be added to the market if $p > f = r$ since g permits would be auctioned and then nothing would be bought back by the price support program; on the other hand, nothing would be added to the market if $p < f = r$ since in that case g permits would be auctioned and then the maximum of g permits would be bought back by the price support program.

3. Simulation results

To illustrate our propositions, we carried out a set of simulations of the three policy regimes. We used Gouel's 2017 very useful tool for specifying, solving and simulating rational expectations equilibria with the Matlab CompEcon tools developed by Miranda and Fackler (2002).¹⁵ The policies of no floor, soft floor and hard floor are implemented as maximum buyback amounts (\bar{g}) of 0, g , or ∞ at floor price $f = 105$. As for the other parameters used in our simulations, we assumed $g = 40$, $\beta = .95$, $\pi_1 = \pi_K = .5$, $\alpha_1 = 50$, $\alpha_K = -50$, and $D(p) = 105 - .75p$.¹⁶

As one might expect, imposing a hard floor far below the equilibrium price will not displace that price and imposing a hard floor far above the equilibrium price will cause the price to rise to the level of the floor. But as our theory implies and our simulations illustrate, there is a considerable range in which a floor inserted below the equilibrium price causes it to jump up. And a floor inserted above the *laissez-faire* price causes it to jump up above the floor. Action at a distance occurs in both cases and is stronger with a hard floor than with a soft floor.

Fig. 2 shows the *laissez-faire* price as a strictly decreasing function of availability. Availability of 36 units induces a *laissez-faire* price of \$105 per unit. Larger availability induces a *laissez-faire* price below the floor and smaller availability induces a *laissez-faire* price above the floor. The figure also depicts the new equilibrium price once a hard or soft floor is introduced at $f = \$105$ per unit.

If the availability is below 12.5 units, the floor of \$105 per unit is so far below the *laissez-faire* price (which is higher than \$122 per unit) that inserting the floor has no effect. But if availability is between 12.5 and 36 units, the floor of \$105 per unit under

¹⁴ In Lee's model, the price ceiling is imposed in every period, but the new equilibrium price path reaches the ceiling only at the end. The new equilibrium price path would therefore be unchanged if instead the ceiling price was known from the outset only to be imposed at the end. Salant et al. (2022) exploits a stochastic version of the same property. They consider a two-period model where the floor binds only in the second period (in the state with the lower price) but never binds in the first period for any floor they consider. In that case, if the floor is imposed only in the second period rather than in both periods, the equilibrium would be unaffected since each market would continue to clear.

¹⁵ For each parameter vector, we first calculate the rational expectations equilibrium decision rule. Then we simulate our model for 200 periods for each of 1,000 independent draws from the distribution of shocks for the 200 periods. We use the same initial availability for all draws.

¹⁶ We had the demand-shock interpretation in mind when selecting these parameters. Note that if the shocks were instead to supply, the realized harvest in state K is negative: $g + \alpha_K = -10$. Had we selected $g = 50$ instead, the results would have been qualitatively unchanged and the realized harvest in state K would have been 0. But we retained this parameterization so that we could call this minor issue to the reader's attention.

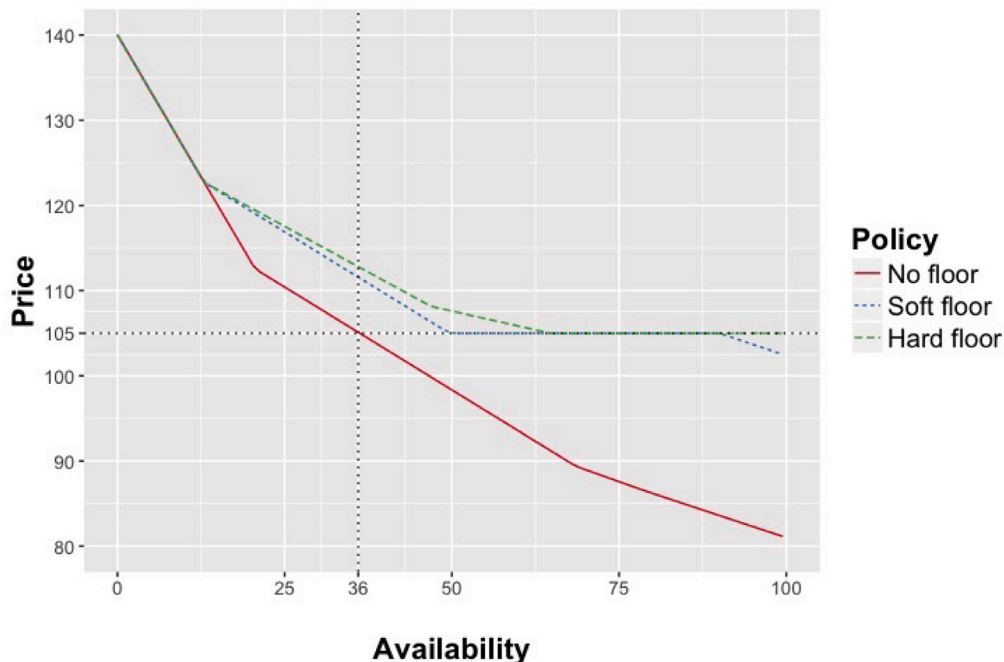


Fig. 2. Action at a distance: imposing a \$105 floor above or below the *laissez-faire* price causes the current-period equilibrium price to rise. The effect is stronger with a hard floor than with a soft floor.

the *laissez-faire* price is close enough that inserting it causes the price to rise: the price with either type of floor strictly exceeds the price with no floor, and a hard floor raises the price by more than a soft floor.

If availability exceeds 62 units, then the floor of \$105 per unit is so far above the *laissez-faire* price that imposing a floor just causes the price to jump up to the level of the floor (\$105 per unit). But if availability is between 36 and 62 units, imposing the \$105 hard floor above the *laissez-faire* price is close enough that it causes the price to jump up so that it strictly exceeds that floor. If availability is between 36 units and 50 units, imposition of a soft floor induces a similar jump above the floor of \$105 per unit although the gap is smaller than with a hard floor.

Thus, for any availability between 12.5 units and 62 units, there is action at a distance when a hard floor is imposed. The equilibrium price exceeds the hard floor, but the floor nonetheless affects that price. Action at a distance also occurs with a soft floor when availability is between 12.5 units and 50 units.¹⁷

In each proposition and corollary as well as in Fig. 2, we have consistently investigated the *impact* on the price of imposing a floor if initial availability is unchanged. It is, however, reasonable to depart once from this approach by asking if action at a distance persists in the very long run if availability is allowed to evolve stochastically to its steady-state distribution after the policy is introduced. It does persist as we show in Fig. 3. We ran 1,000 simulations of the model at different price floors and calculated the mean price for each policy regime.

Fig. 3 gives the results for four different price floors: \$55, \$80, \$100 and \$110 per unit.¹⁸ The *laissez-faire* price fluctuates between \$80 per unit and \$90 per unit. So the first two floors are below the *laissez-faire* price and the second two floors are above it. The floor of \$55 per unit is so far below the *laissez-faire* price that it has no effect. The floor of \$80 per unit, whether hard or soft, raises

¹⁷ If one increases availability until equilibrium price with a hard or soft floor just reaches the floor, the price function has a kink. There is no “smooth-pasting” result like that in Krugman (1991), where the exchange rate is tangent to the hard floor. Our model differs from his in many respects. In particular, our asset is depleted through true-ups (or consumption) and is restored through permit auctions (or harvests). Moreover, our fundamental (availability) unlike that of Krugman (velocity) does not execute a random walk when the price exceeds the hard floor because our demand is price sensitive. Finally, we derived our price functions from optimizing behavior and he did not. It turns out that none of this matters for the smooth-pasting result. Our model is in discrete time and his is in continuous time. Once we consider the continuous-time limit of our model, the smooth-pasting result emerges. In particular, we assumed $D(p; h) = (105 - .75p)h; g(h) = 40h; \beta(h) = (1 + .05h)^{-1}; \pi_1 = \pi_K = .5; \alpha_1 = 50h$ and $\alpha_K(h) = -50h$. As $h \rightarrow 0$ $p^H(A; h)$ and $p^S(A; h)$ exhibit smooth pasting. See Appendix C.

¹⁸ In Fig. 3, we zoom in on periods 50 through 100 of the 200 periods simulated to better show inter-period variability.

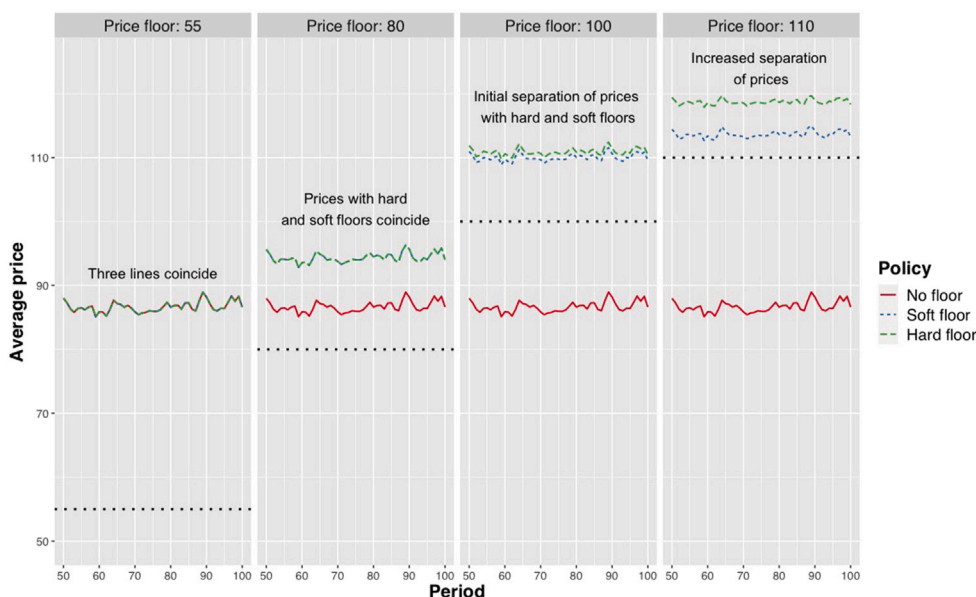


Fig. 3. The long-run effect on average commodity prices for a price floor imposed on either side of the laissez-faire price. The black dotted lines indicate the floor price imposed. (Only periods 50 - 100 shown.).

the price above the floor but by the same amount. The floor of \$100 per unit or \$110 per unit raises the price to a level which exceeds the floor and the hard floor has the stronger effect.¹⁹

4. Conclusions

Price floors are a common form of policy intervention to bolster prices. In introductory economics textbooks, minimum wages in labor markets and price supports in grain markets are often the most common examples. In either case, it is argued, (1) a floor imposed below the *laissez-faire* price will have no effect while (2) a floor imposed above it will raise the price until it coincides with the floor. Each proposition is valid when applied to the labor market but, as we show, not when applied to storable assets like grains or pollution allowances.

In this paper, we prove that whether a floor is inserted above or below the *laissez-faire* price of a storable asset, it may immediately raise the price, leaving it strictly above the floor. Despite the gap that results, the floor may nonetheless influence the price, a phenomenon we dub “action at a distance”. Our paper also shows that a seemingly nonbinding hard floor will raise prices by strictly more than a seemingly nonbinding soft floor. Conversely, removing a price floor may cause an immediate drop in the price—possibly even to a level below where the floor had been.

To derive these results, we develop a unified theoretical framework encompassing hard floors, soft floors, and *laissez-faire* and combine it with a versatile contraction-mapping approach that allows us to capture rational price expectations in an infinite-horizon, stochastic setting. We use our framework to compare the effect on price of either type of floor. We provide a condition sufficient for a hard floor to raise the price by strictly more than a soft floor set at the same level. We show by simulation that our general results on the effects of hard and soft floors persist in the long run for the specific demand curve and probability distribution used in the simulation.

A soft floor occurs whenever there is a limit (budgetary or otherwise) on the scale of government intervention in the market. In the case of cap-and-trade programs, it arises whenever permit auctions have a reserve price as they typically do in the US. Permit auctions in Europe’s carbon market (EU-ETS) do not currently have auction reserve prices so our theoretical analysis may be useful in guiding econometric analysis of how soft floors affect permit markets, resulting in improved policy. In addition, our findings should eliminate the unfortunate current practice of analyzing price collars in emissions-trading markets as if they were hard floors when in fact they are soft floors.

Our results illuminate observations from disparate fields. They show that Krugman’s insight about smooth pasting in foreign exchange markets with hard floors extends to markets for other storable assets with markedly different characteristics and can be given a micro-foundation. Our analysis also explains the intuition in Borenstein et al. (2019) that the “nonbinding” soft floor under the permit price in California’s cap-and-trade program (AB-32) is raising its price. Finally, they provide a rationale for the observation in the *Financial Times* (2013) about the influence of the hard floor in China’s rice market.

¹⁹ Action at a distance is evident in Fig. 3 between a floor of 80 and a floor of 110. Discussion of the full range of floors where it occurs is discussed in Appendix D.

Appendix A. General equilibrium interpretation

Readers unfamiliar with the literature on agricultural carryovers may prefer a general equilibrium interpretation of the standard model before reading our paper. To accommodate such readers, we discuss here a two-period carryover model with uncertainty about the future grain endowment. As we show, introducing a “background good” which generates constant marginal utility provides a simple general equilibrium model with the properties assumed in the carryover literature. Although we limit our treatment to two periods with two states in the second period, the formulation is easily generalized.

Suppose price-taking consumers are identical in their preferences, beliefs, and endowments. Let the representative agent consume (z, z^h, z^l) units of grain and (y, y^h, y^l) units of background good in the first period and in the two possible states of the second period. Assume he is endowed with $(\bar{z}, \bar{z}^h, \bar{z}^l)$ units of grain and $(\bar{y}, \bar{y}^h, \bar{y}^l)$ units of background good. Assume grain may be costlessly carried from the first to the second period but the background good is not storable. Denote the grain carryover as $x \geq 0$. Assume the representative agent assigns probabilities π_h and π_l to the two states, where $\pi_h + \pi_l = 1$. The six consumptions, six endowments, and two probabilities are nonnegative. Assume consuming z units of grain and y units of the background good provides quasilinear utility $f(z) + \alpha y$ in period 1 and the corresponding utility in each state of the second period, where $f(\cdot)$ is strictly increasing and strictly concave. Assume that $\alpha > 0$, and that second period utilities are discounted by $\beta < 1$.

To avoid having to consider corners, where consumption of grain or the background good is zero, we assume that $f(\cdot)$ satisfies the Inada condition and that the endowment of the background good in the first period and in each state of the second period is strictly positive.²⁰ We must still consider, however, the possibility that the carryover is zero. Then the agent solves:

$$\max_{x \geq 0, y, y^h, y^l, z, z^h, z^l > 0} f(z) + \alpha y + \beta (\pi_h (f(z^h) + \alpha y^h) + \pi_l (f(z^l) + \alpha y^l)) \tag{6}$$

subject to:

$$p_y y + p_z z \leq p_y \bar{y} + p_z (\bar{z} - x) \tag{7}$$

$$p_y^h y^h + p_z^h z^h \leq p_y^h \bar{y}^h + p_z^h (\bar{z}^h + x) \tag{8}$$

$$p_y^l y^l + p_z^l z^l \leq p_y^l \bar{y}^l + p_z^l (\bar{z}^l + x). \tag{9}$$

Assign the multipliers λ , λ_h , and λ_l respectively to the three constraints. Given our assumptions, the constraints bind and the multipliers are strictly positive. The following conditions must hold at the optimum:

$$f'(z) - \lambda p_z = 0 \tag{10}$$

$$\alpha - \lambda p_y = 0 \tag{11}$$

$$\beta \pi_h f'(z^h) - \lambda_h p_z^h = 0 \tag{12}$$

$$\beta \pi_h \alpha - \lambda_h p_y^h = 0 \tag{13}$$

$$\beta \pi_l f'(z^l) - \lambda_l p_z^l = 0 \tag{14}$$

$$\beta \pi_l \alpha - \lambda_l p_y^l = 0 \tag{15}$$

$$x \geq 0, \lambda_h p_z^h + \lambda_l p_z^l - \lambda p_z \leq 0, \text{ complementary slackness.} \tag{16}$$

As shown below, these first-order conditions imply that (i) demand for z depends only on the contemporaneous relative price and is independent of the value of the endowment; (ii) the carryover is zero if the current price exceeds expected price next period discounted by β ; (iii) if the carryover is strictly positive then the current price must equal the expected price next period discounted by β .

Demand for Grain

Eliminating λ from (10) and (11), yields (17). Eliminating λ_h from (12) and (13) yields (18). Eliminating λ_l from (14) and (15) yields (19).

$$f'(z) = p_z \left(\frac{\alpha}{p_y} \right) \tag{17}$$

$$f'(z^h) = p_z^h \left(\frac{\alpha}{p_y^h} \right) \tag{18}$$

$$f'(z^l) = p_z^l \left(\frac{\alpha}{p_y^l} \right). \tag{19}$$

²⁰ The Inada condition ensures that the agent will consume *some* grain regardless of its relative price. In contrast, zero consumption of the background good can occur at some relative prices. But, in equilibrium, consumption of the background good must be strictly positive in the first period and in each state of the second period. This follows since (1) the three endowments of the background good are strictly positive, (2) the good cannot be stored between periods, and (3) agents are identical and, given quasiconcavity, will make identical choices. To simplify the exposition of what occurs in equilibrium, therefore, we ignore cases where the background good is not consumed.

Hence, the grain demand depends on the contemporaneous relative price. As long as relative prices do not change, any increase in wealth results in the agent choosing to consume more of the background good and the same amounts of grain. Denote the relative prices by “hats”:

$$\hat{p} = \frac{p_z}{p_y} \tag{20}$$

$$\hat{p}_h = \frac{p_z^h}{p_y^h} \tag{21}$$

$$\hat{p}_l = \frac{p_z^l}{p_y^l} \tag{22}$$

Carryover Depends on Current and Future Prices

Using (11), (13), and (15) to eliminate the three multipliers, (16) can be re-written as:

$$x \geq 0, \beta[\pi_l \hat{p}_l + \pi_h \hat{p}_h] - \hat{p} \leq 0, \text{ with complementary slackness.} \tag{23}$$

Equilibrium Prices and Storage

Since agents are identical, the relative prices $(\hat{p}, \hat{p}_h, \hat{p}_l)$ must adjust so that the desired consumption of grain in the first period equals the supply of grain minus the carryover; the desired consumption of grain in state i ($i = h, l$) equals the state-contingent harvest plus the carryover; and given the current price and the expected price next period discounted by the exogenous parameter β . The four endogenous variables $(\hat{p}, \hat{p}_h, \hat{p}_l, x)$ are determined by the following four equations:

$$\hat{p} = \frac{1}{\alpha} f'(\bar{z} - x) \tag{24}$$

$$\hat{p}_h = \frac{1}{\alpha} f'(\bar{z}^h + x) \tag{25}$$

$$\hat{p}_l = \frac{1}{\alpha} f'(\bar{z}^l + x) \tag{26}$$

$$x \geq 0, \beta[\pi_h \hat{p}_h + \pi_l \hat{p}_l] - \hat{p} \leq 0, \text{ complementary slackness.} \tag{27}$$

Appendix B. Proposition 2

Proposition 2. Under each of the three policies, there exists a unique equilibrium price function $p^i(A)$, which is continuous. The price functions can be weakly ranked across the three policies: $p^H(A) \geq p^S(A) \geq p^N(A)$. If the following conditions hold: (i) $x^N(\bar{A}) > 0$; (ii) $p^N(\bar{A}) > f$; (iii) $p^S(x^S(\bar{A}) + g + \alpha_1) < f$, then $p^H(\bar{A}) > p^S(\bar{A}) > p^N(\bar{A}) > f$ and $x^H(\bar{A}) > x^S(\bar{A}) > x^N(\bar{A})$.

Proof. The proof of Proposition 2 proceeds in three steps. We show (1) there exists a unique, continuous, bounded, weakly decreasing price function $p^i(A)$ for each policy; (2) these price functions can be weakly ranked for any availability A as $p^H(A) \geq p^S(A) \geq p^N(A)$; and (3) for any availability \bar{A} and price floor f satisfying conditions (i)-(iii), $p^H(\bar{A}) > p^S(\bar{A}) > p^N(\bar{A}) > f$ and $x^H(\bar{A}) > x^S(\bar{A}) > x^N(\bar{A})$.

We again denote the government demand correspondence as $R(p_i; \bar{g})$ and again use the compact notation $R^i(p_i^j)$ defined after equation (1):

$$R(p_i; \bar{g}) \begin{cases} = 0 & \text{if } p_i > f \\ \in [0, \bar{g}] & \text{if } p_i = f \\ = \bar{g} & \text{if } p_i < f. \end{cases} \tag{28}$$

Step 1.

We review the application to equilibrium price functions of contraction-mapping arguments usually applied to value functions in infinite horizon dynamic programming.²¹ Suppose we know the price function under policy i , $p^i_j(A)$, in some period j and want to compute the price function one period earlier, $p^i_{j+1}(A)$. Suppose $p^i_j(A)$ is continuous, bounded and weakly decreasing. The market must clear in period $j + 1$ so the following equation must hold:

$$A = x^i_{j+1} + D(p^i_{j+1}) + R^i(p^i_{j+1}). \tag{29}$$

Moreover, profit-maximization requires:

$$x^i_{j+1} \geq 0, p^i_{j+1} - \beta \sum_{k=1}^K \pi_k p^i_j(x^i_{j+1} + g + \alpha_k) \geq 0, \text{ with complementary slackness.} \tag{30}$$

²¹ For an introductory discussion of contraction-mapping arguments applied to the value function in dynamic programming, see Adda and Cooper (2003) and its references to the more rigorous exposition in Stokey and Lucas (1989). For an application of contraction-mapping arguments to the equilibrium price function of an infinite-horizon storage model, see Salant (1983).

These equations determine the unknowns x_{j+1}^i and p_{j+1}^i under policy i as functions of A . Denote the procedure of deducing $p_{j+1}^i(A)$ from $p_j^i(A)$ as the mapping $T^i : p_{j+1}^i(A) = T^i p_j^i(A)$, for $i = H, S, N$.

It is easily verified that $p_{j+1}^i(A)$ will also be continuous, bounded, and weakly decreasing.²² Moreover, for policy i , one can verify from (29) and (30) that the mapping T^i satisfies Blackwell's sufficient conditions for a contraction mapping of modulus $\beta \in [0, 1]$.²³ Since the space of continuous, bounded, weakly decreasing functions is a complete metric space under the supremum norm, it follows that each of the three mappings has its own unique fixed point $p^i(A) = T^i p^i(A)$.

Moreover, since T^i contracts, we can start with any continuous, bounded, weakly decreasing function, apply T^i to it repeatedly and in this way generate a sequence of continuous, bounded, weakly decreasing functions that converges uniformly to the fixed point of T^i . The fixed point will also be continuous, bounded and weakly decreasing.

Step 2.

We now show that the three limit functions can be weakly ranked: $p^H(A) \geq p^S(A) \geq p^N(A)$. We do so by showing that if a trio of continuous, bounded, weakly decreasing functions can be weakly ranked as $p_j^H(A) \geq p_j^S(A) \geq p_j^N(A)$ for all $A \geq 0$ and $j = 1, \dots$ then $p_{j+1}^H(A) \geq p_{j+1}^S(A) \geq p_{j+1}^N(A)$ for all A .²⁴

Proof. To show that $p_{j+1}^H(A) \geq p_{j+1}^S(A)$ for all A , we first suppose the contrary: that $p_{j+1}^S > p_{j+1}^H$ for some A . Consider first an A so low that $p_{j+1}^S > f$. In that case, (28) implies $R^S = 0$ and so (29) requires that $x_{j+1}^S > x_{j+1}^H \geq 0$. But if $x_{j+1}^S > 0$, then (30) requires that $p_{j+1}^S = \beta \sum_{k=1}^K \pi_k p_j^S(x_{j+1}^S + g + \alpha_k) > p_{j+1}^H \geq \beta \sum_{k=1}^K \pi_k p_j^H(x_{j+1}^H + g + \alpha_k)$, where we have used our hypothesis that $p_{j+1}^S > p_{j+1}^H$. But this deduction that $\beta \sum_{k=1}^K \pi_k p_j^S(x_{j+1}^S + g + \alpha_k) > \beta \sum_{k=1}^K \pi_k p_j^H(x_{j+1}^H + g + \alpha_k) \geq \beta \sum_{k=1}^K \pi_k p_j^H(x_{j+1}^S + g + \alpha_k)$ contradicts our assumption that $p_j^H(A) \geq p_j^S(A)$ for all A and that both functions are weakly decreasing.

Consider next an A sufficiently high that $p_{j+1}^S \leq f$. Then our hypothesis that $p_{j+1}^S > p_{j+1}^H$ implies $p_{j+1}^H < f$. But this contradicts (28), since under a hard floor the price never falls strictly below the floor.

Since the hypothesis that $p_{j+1}^S > p_{j+1}^H$ holds for no A , we conclude that $p_{j+1}^H(A) \geq p_{j+1}^S(A)$ for all A .

To show that $p_{j+1}^S(A) \geq p_{j+1}^N(A)$ for all A , suppose to the contrary that, for some A , $p_{j+1}^S(A) < p_{j+1}^N(A)$.

Note that independent of whether $p_{j+1}^S > f$ or $p_{j+1}^S \leq f$, (28) implies $R^S \geq R^N = 0$ and so (29) requires that $x_{j+1}^N > x_{j+1}^S \geq 0$. But if $x_{j+1}^N > 0$, then (30) requires that $p_{j+1}^N = \beta \sum_{k=1}^K \pi_k p_j^N(x_{j+1}^N + g + \alpha_k) > p_{j+1}^S \geq \beta \sum_{k=1}^K \pi_k p_j^S(x_{j+1}^S + g + \alpha_k)$, where we have used our hypothesis that $p_{j+1}^N > p_{j+1}^S$. But this deduction that $\beta \sum_{k=1}^K \pi_k p_j^N(x_{j+1}^N + g + \alpha_k) > \beta \sum_{k=1}^K \pi_k p_j^S(x_{j+1}^S + g + \alpha_k) \geq \beta \sum_{k=1}^K \pi_k p_j^S(x_{j+1}^N + g + \alpha_k)$ contradicts our assumption that $p_j^S(A) \geq p_j^N(A)$ for all A and that both functions are weakly decreasing.

Since the hypothesis that $p_{j+1}^N > p_{j+1}^S$ holds for no A , we conclude $p^S(A) \geq p^N(A)$ for all A .

Hence, $p_{j+1}^H(A) \geq p_{j+1}^S(A) \geq p_{j+1}^N(A)$ for all A . □

Step 3.

If the following conditions hold: (i) $x^N(\bar{A}) > 0$; (ii) $p^N(\bar{A}) > f$; and (iii) $p^S(x^S(\bar{A}) + g + \alpha_1) < f$, then $p^H(\bar{A}) > p^S(\bar{A}) > p^N(\bar{A}) > f$.

Suppose current availability is \bar{A} . Given condition (iii) and (28), it is easy to see that $R^S(p^S(x^S(\bar{A}) + g + \alpha_1)) = g$. Given condition (iii) and the weak ranking of the price functions for any A ,

$$p^H(x^S(\bar{A}) + g + \alpha_1) > p^S(x^S(\bar{A}) + g + \alpha_1) \geq p^N(x^S(\bar{A}) + g + \alpha_1), \tag{31}$$

where the strict inequality follows since the price is never below f when the floor is hard.

To show that the last inequality must also be strict, we rule out the alternative: $p^S(x^S(\bar{A}) + g + \alpha_1) = p^N(x^S(\bar{A}) + g + \alpha_1)$. Under the two policies, if the same supply were available ($x^S(\bar{A}) + g + \alpha_1$) and consumers paid the same price, then demand would be the same, but the government would purchase g units only in the case of the soft floor: $R^S(p^S(x^S(\bar{A}) + g + \alpha_1)) = g > R^N = 0$. Market clearing (29) then implies that $x^N(x^S(\bar{A}) + g + \alpha_1) > x^S(x^S(\bar{A}) + g + \alpha_1) \geq 0$. Therefore, complementary slackness (30) implies that:

$$p^N(x^S(\bar{A}) + g + \alpha_1) = \beta \sum_{k=1}^K \pi_k p^N(x^N(x^S(\bar{A}) + g + \alpha_1) + g + \alpha_k) \tag{32}$$

$$= p^S(x^S(\bar{A}) + g + \alpha_1) \geq \beta \sum_{k=1}^K \pi_k p^S(x^S(x^S(\bar{A}) + g + \alpha_1) + g + \alpha_k). \tag{33}$$

²² $p_{j+1}^i(A)$ is bounded in $[0, p^i]$; continuity follows from the implicit function theorem. If $p_{j+1}^i(A)$ were strictly increasing (instead of weakly decreasing), an increase in A would cause strict excess supply in (29) and would necessitate a strict increase in x_{j+1}^i . But the strict increase in both the price and carryover would violate (30).

²³ To verify his monotonicity condition, note that if the price function at j were uniformly weakly higher, the discounted expected price at j would be weakly higher for any A and the current price would again be uniformly weakly higher provided $x_{j+1}^i > 0$. If instead $x_{j+1}^i = 0$, there would be no change in the price at $j + 1$ (and hence it would be trivially weakly higher). To verify Blackwell's discounting condition, note that if a constant $C \geq 0$ were added to $p_j^i(A)$, the expected price would rise by βC and therefore p_{j+1}^i would rise by βC if $x_{j+1}^i > 0$ and would not rise at all otherwise.

²⁴ This establishes that starting with $p_1^H(A) \geq p_1^S(A) \geq p_1^N(A)$ and applying T^H repeatedly to the first initial function, T^S repeatedly to the second initial function and T^N repeatedly to the third initial function results in limit functions which inherit the same weak ranking. But because each operator is a contraction mapping, we have established much more. We would have converged to the same trio of weakly ranked limit functions if instead we had started with any trio of continuous, bounded, weakly decreasing initial functions—say, for example, functions with the opposite ranking $p_1^H(A) < p_1^S(A) < p_1^N(A)$. Thus, while the ranking of initial functions we chose is convenient, the weak ranking of the limit functions is independent of the choice of initial functions.

But since $x^N(x^S(\tilde{A}) + g + \alpha_1) > x^S(x^S(\tilde{A}) + g + \alpha_1) \geq 0$ and the literature has established that $p^N(\cdot)$ is strictly decreasing, this would imply that

$$\sum_{k=1}^K \pi_k p^N(x^S(x^S(\tilde{A}) + g + \alpha_1) + g + \alpha_k) > \sum_{k=1}^K \pi_k p^S(x^S(x^S(\tilde{A}) + g + \alpha_1) + g + \alpha_k),$$

which cannot occur since in every state k the price under the soft floor will be weakly higher when the same stock is available. Therefore, the second inequality in (31) must be strict. Since the price functions can be weakly ranked when evaluated at the same availability and, as we have just established, can be *strictly* ranked in state 1 under the conditions of Proposition 2, we conclude:

$$\sum_{k=1}^K \pi_k p^H(x^S(\tilde{A}) + g + \alpha_k) > \sum_{k=1}^K \pi_k p^S(x^S(\tilde{A}) + g + \alpha_k) > \sum_{k=1}^K \pi_k p^N(x^S(\tilde{A}) + g + \alpha_k). \tag{34}$$

Suppose current availability is \tilde{A} and condition ii holds. Since the price functions can be weakly ranked, $p^H(\tilde{A}) \geq p^S(\tilde{A}) \geq p^N(\tilde{A}) > f$. Hence, (28) implies that $R^H(p^H(\tilde{A})) = R^S(p^S(\tilde{A})) = R^N = 0$. Market clearing (29) implies that $x^H(\tilde{A}) \geq x^S(\tilde{A}) \geq x^N(\tilde{A}) > 0$, where the strict inequality is assumed in condition i.

These carryovers can, in fact, be *strictly* ranked. For, consider first the alternative $x^H(\tilde{A}) = x^S(\tilde{A}) > 0$. Then $p^H(\tilde{A}) = p^S(\tilde{A}) = \beta \sum_{k=1}^K \pi_k p^H(x^H(\tilde{A}) + g + \alpha_k) = \beta \sum_{k=1}^K \pi_k p^S(x^S(\tilde{A}) + g + \alpha_k) = \beta \sum_{k=1}^K \pi_k p^H(x^S(\tilde{A}) + g + \alpha_k)$. The equality of the prices comes from (29), the second and third equalities from (30) and the last from our hypothesis that the carryover under the two policies is equal. But since the last equality violates (34), the hypothesis must be false and instead $x^H(\tilde{A}) > x^S(\tilde{A})$.

Consider next the alternative $x^N(\tilde{A}) = x^S(\tilde{A}) > 0$, where the second inequality is assumed in condition i. Then $p^N(\tilde{A}) = p^S(\tilde{A}) = \beta \sum_{k=1}^K \pi_k p^N(x^N(\tilde{A}) + g + \alpha_k) = \beta \sum_{k=1}^K \pi_k p^S(x^S(\tilde{A}) + g + \alpha_k) = \beta \sum_{k=1}^K \pi_k p^N(x^S(\tilde{A}) + g + \alpha_k)$. The equality of prices comes from (29), the second and third equalities from (30), and the last from our hypothesis that the carryover under the two policies is equal. But the last equality violates (34). So the hypothesis must be false and $x^S(\tilde{A}) > x^N(\tilde{A})$.

Conditions (i)-(iii) of Proposition 2, therefore, have the following implications: $x^H(\tilde{A}) > x^S(\tilde{A}) > x^N(\tilde{A}) > 0$ and, from market clearing (29), $p^H(\tilde{A}) > p^S(\tilde{A}) > p^N(\tilde{A})$ as was to be proved. \square

Appendix C. Smooth pasting

In Fig. 2, if one increases availability until equilibrium price with a hard or soft floor just reaches the floor, the price function has a kink. There is no “smooth-pasting” result like that in Krugman (1991), where the exchange rate is tangent to the hard floor. Our model differs from his in many respects. In particular, our asset is depleted through true-ups (or consumption) and is restored through permit auctions (or harvests). Moreover, our fundamental (availability) unlike that of Krugman (velocity) does not execute a random walk when the price exceeds the hard floor because our demand is price sensitive. Finally, we derived our price functions from optimizing behavior and he did not. Moreover, we conducted our analysis in discrete time and he conducted his in continuous time. Which of these differences accounts for the absence of smooth pasting in our model?

To investigate further, we simulated our infinite-horizon discrete-time model where 1 period equals 1 year.²⁵ We then treated 1 period as a fraction $h \leq 1$ of a year. For example, if $h = 1/365$, each period would represent 1 day. In Fig. C.1, the left-hand panel depicts $p^H(A)$ when the time interval between auctions is 365 days while the panel on the right shrinks the interval to 3.65 days.²⁶ As the two panels of Fig. C.1 reflect, smooth pasting appears in our model in the continuous-time limit.

Appendix D. The range of floors generating action at a distance

Although we have provided a simulation illustrating that action at a distance may occur over a broad range of floors, readers may be interested in the factors that determine this range. This is difficult to explain in an infinite-horizon model. Accordingly, we consider here a more tractable case: a two-period model where the representative consumer is endowed with A units of grain in the first period and none in the second; however, grain is storable and the agent knows the probability that one of K harvests will arrive next period. It is optimal to sell one’s holdings (the realized harvest plus anything carried over from the first period) in the second period because stocks carried beyond that are worthless. Hence, the price realized in the second period can be determined by substituting the carryover plus the state-dependent harvest into the inverse demand curve; the first-period price can be determined by deducting from A the amount of grain carried into the second period.

Assume that in the absence of a carryover, the expected price next period, discounted by the exogenous factor $1/(1+r)$, would exceed the price if the initial available stock was all put on the market in the first period. Then in the no-policy equilibrium, there would be a positive carryover. Assume a hard floor at level f is imposed in both periods. In the absence of a floor, denote the initial price as p^N and the second-period price in the state with the largest harvest next period as p_1^N .

²⁵ We thank David K. Levine for suggesting this exercise.

²⁶ We assumed $D(p; h) = (105 - .75p)h$; $g(h) = 40h$; $\beta(h) = (1 + .05h)^{-1}$; $\pi_1 = \pi_K = .5$; $\alpha_1 = 50h$ and $\alpha_K(h) = -50h$. As $h \rightarrow 0$, the kinks at the points where $p^H(A; h)$ and $p^S(A; h)$ first reach the floor disappear.

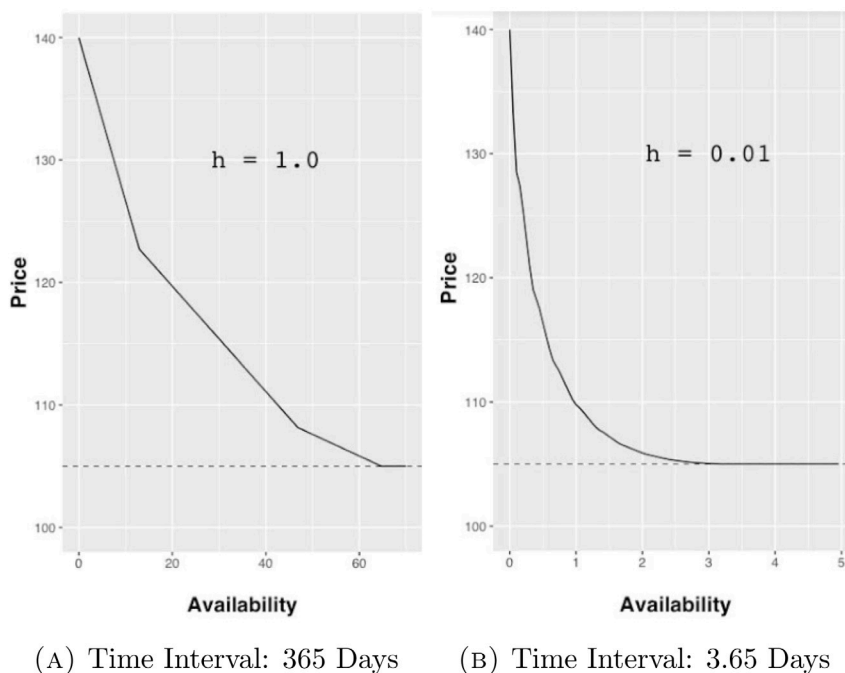


Fig. C.1. Smooth pasting arises as the time interval shrinks.

A hard floor set below p_1^N will not displace the equilibrium. A floor set any larger than $f^* = p_1^N$, will induce an increased carryover and a higher initial price.²⁷ Hence, action at a distance begins at $f = f^* = p_1^N$. Note the implication: even if the chance next period of the lowest price (largest harvest) is very small, action at a distance begins if a floor is imposed at that lowest price.

Although the first-period price is initially above the hard floor f and although that price increases when f increases, eventually the floor overtakes the first-period price. Only then does the gap between the first-period price and the hard floor disappear. Denote that floor as f^{**} . Action at a distance ends at f^{**} .

At $f = f^{**}$, the expected discounted price next period is the probability-weighted average of the discounted prices next period. At $f = f^{**}$, some of the K states will have harvests so large (and prices so low) that government intervention is called for, driving the discounted price in each such state up to $f^{**}/(1+r)$. The remaining states have harvests so small (and prices so high) that no government intervention is authorized. $f^{**}/(1+r)$, weighted by the probability that some low-price state occurs (denoted Π_L) plus the expected discounted price conditional on no government support (denoted E) weighted by the complementary probability $(1 - \Pi_L)$ must equal the first period price of f^{**} . Hence the mean price in these states without intervention must exceed $f^{**}(1+r)$. In particular, $E = f^{**} \left(1 + \frac{r}{(1-\Pi_L)}\right) > f^{**}(1+r)$.

Salant et al. (2022) works out a similar example with $K = 2$, $r = 0$, but with demand uncertainty (50% low, 50% high) instead of harvest uncertainty. Using linear demand, the no-policy price in the first period is $p^N = 152$. Action at a distance begins at $f^* = 92$ and ends at $f^{**} = 182$.

Appendix E. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.eurocorev.2022.104122>.

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²⁷ To see why, note that if the carryover did not change in response to a higher floor, then the price in the first period would not change nor would the price change in any second-period state where the price exceeded the larger floor. But in every other state, the larger floor would result in a higher price and hence the expected discounted price in the second period would exceed the first-period price, when before the floor increased, the two had been equal. This profit-opportunity is what motivates the agent to enlarge his carryover, which in turn raises the first-period price.

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